Midterm – Mat1190.1 Wednesday, May 31, 2006

Name Key   Total Points = 107

1) Solve for x (12 points).

a) $\frac{5}{3}(x+2)=\frac{1}{2}-x$, $\frac{5x}{3}+\frac{10}{3}=\frac{1}{2}-x$

$\frac{8x}{3}=\frac{1}{2}-\frac{10}{3}=\frac{3-20}{6}=\frac{-17}{6}$

$8x=-\frac{17}{2} \Rightarrow \boxed{x=\frac{-17}{16}}$

b) $x^2-x-20=0$

$(x+4)(x-5)=0$

$\boxed{x=-4}$, $\boxed{x=5}$

c) $10^x=0.001$

$10^x=10^{-3}$, $\boxed{x=-3}$

d) $\log_3(x)=\frac{1}{81}$

$x=(3)^{1/81}$ OR $\boxed{x=\sqrt[81]{3}}$

2) What is the inverse of $f=\{(1,1),(2,5),(3,3),(8,9)\}$? ______________

(4 points) $f^{-1}(x)=\{(1,1),(5,2),(3,3),(9,8)\}$.

3) Given $f(x)=-x^3+7$ and $g(x)=2x^2-3$, solve for x in each case. (6 points)

a) $f(x)=0$, $-x^3+7=0 \Rightarrow x^3=7$

$x=(7)^{1/3}$, $\boxed{x=\sqrt[3]{7}}$

b) $g(x)=0$ $=2x^2-3 \Rightarrow x^2=\frac{3}{2}$

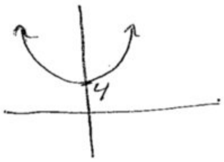
$\boxed{x=\pm\sqrt{\frac{3}{2}}}$

4) State the domain and range of each of the following functions (16 points).

a) $2x+5$ Domain $=\mathbb{R}$

Range $=\mathbb{R}$

b) x^2+4



Domain $=\mathbb{R}$

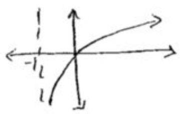
Range $=[4,\infty)$.

c) $\frac{7}{\sqrt{3x-4}}$, Domain : $3x-4>0$, $\boxed{x > \frac{4}{3}}$

$(\frac{4}{3}, \infty)$.

$\boxed{\text{Range} : (0, \infty)}$

d) $\ln(x+1)$, Domain : $x+1>0$, $\boxed{x > -1}$

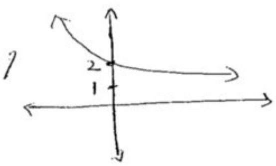


Range : $\mathbb{R}$.

5) What is the value (if any) of the following (6 points)?

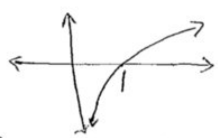
a) $(e^{-x}+1)$ as $x \to -\infty$: $e^{-x}+1 = \left(\frac{1}{e}\right)^x + 1$

$= \infty$



b) $\log_5(x)$ as $x \to \infty$

$= \infty$



6) Given $p(x) = -x+7$ and $s(x) = 2x^2 - x$, evaluate and simplify. (9 points)

a) $s(-1) = 2(-1)^2 - (-1) = 2 + 1 = \boxed{3}$

b) $p(x+1) = -(x+1)+7 = -x-1+7 = \boxed{-x+6}$

c) $s\left(\frac{1}{2x-5}\right) = 2\left(\frac{1}{2x-5}\right)^2 - \left(\frac{1}{2x-5}\right) = \frac{2-(2x-5)}{(2x-5)^2}$

$2 - 2x + 5$

$= \boxed{\frac{7-2x}{(2x-5)^2}}$

7) Given the population function $P = 23(1.0975)^t$ where t is in years and $e^{.093} = 1.0975$, determine the: (6 points)

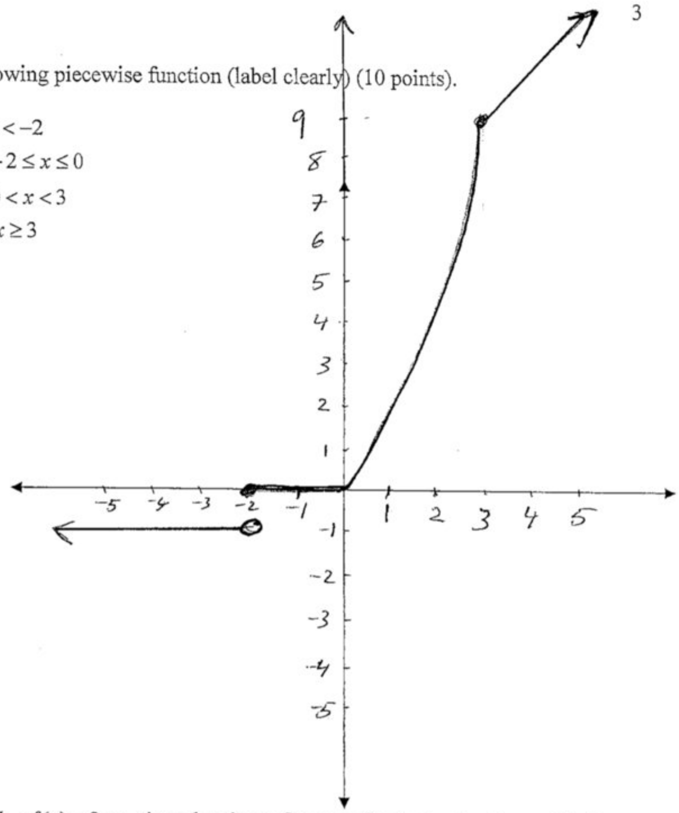
a) percent annual growth rate $0.0975 = \boxed{9.75\%}$

b) percent continuous growth rate

$0.093 = \boxed{9.3\%}$

8) Graph the following piecewise function (label clearly) (10 points).

$$f(x)=\begin{cases}-1, & x<-2\\ 0, & -2\le x\le 0\\ x^2, & 0<x<3\\ 3x, & x\ge 3\end{cases}$$



9) The formula $C=f(r)=2\pi r$ gives the circumference of a circle of radius r. Find: (6 points)

$r=\frac{C}{2\pi}=f^{-1}(C)$

a) $f^{-1}(C)$ $=\frac{C}{2\pi}$

b) $f^{-1}(10\pi)$ $=\frac{10\pi}{2\pi}=5$

10) Determine if each table of values represents a function that is concave up, concave down or neither? Show your work and give reasons. (8 points)

a)

X	0	1	2
Y	7	7	7

constant function
no concavity.

b)

X	0	1	2
Y	1	2	5

Δy: 1 3

$\frac{\Delta y}{\Delta x}$ is increasing, so concave up.

11) Solve for t (6 points).

a) $t\ln(e^5) = 2t + 5$

$t(\ln e^5 - 2) = 5$

$t(5-2) = 5$

$3t = 5$

$\boxed{t = 5/3}$

b) $40e^{0.20t} = 50(1.5)^t$

$\ln 4 + 0.20t = \ln 5 + t\ln(1.5)$

$t[0.20 - \ln(1.5)] = \ln 5 - \ln 4$

$\boxed{t = \dfrac{\ln(5/4)}{0.2 - \ln(1.5)}}$

12) Suppose $f(-3) = \frac{5}{8}$ and $f(2) = 20$. Find a formula for f assuming it is:
(8 points)

a)Linear $m = \dfrac{20 - 5/8}{2-(-3)} = \dfrac{31}{8}$

$y = \frac{31}{8}x + b$

$\frac{31}{\cancel{8}_4}(\cancel{2}) + b = 20$

$b = 20 - \frac{31}{4}$

$b = \frac{49}{4}$

b) Exponential

$y = a(b)^x$

$\frac{5}{8} = a(b)^{-3}$

$20 = a(b)^2$

$\Rightarrow \dfrac{20}{5/8} = \dfrac{ab^2}{ab^{-3}} \Rightarrow 32 = b^5 \Rightarrow b = 2$

$20 = a(2)^2 = a(4) \Rightarrow a = 5$

13) The US census projects the population of the state of Washington using the function $N(t) = 5.4e^{0.013t}$, where $N(t)$ is in millions and t is in years since 1995. (10 points)

a) What is the population's continuous growth rate?

$1.3\% = 0.013$

b) What is the population of Washington in the year 2000?

$N(5) = 5.4\,e^{0.013(5)}$

c) How many years is it before the population triples?

$5.4\,e^{0.013t} = 3(5.4)$

Solve for t.

$e^{0.013t} = 3 \Rightarrow \boxed{t = \dfrac{\ln 3}{0.013}} \approx 84.5$ years.